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DOES COUPLING TO GRAVITY
PRESERVE INTEGRABILITY ?

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ABSTRACT

I describe evidence that the (two-dimensional) integrable chiral Gross-Neveu model might remain integrable when coupled to gravity. These are notes based on a lecture given at the Cargese summer school 1995. The results were obtained in collaboration with Ian Kogan.

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1. Introduction

These notes are based on results obtained in collaboration with Ian Kogan. They have already been briefly announced in our previous articles [1]. Here I will give some more details.

When a renormalisable quantum field theory is coupled to gravity three things may happen: the coupled system is more difficult or even non-renormalisable, it is simpler, or about the same. The first option is of course the generic one in four dimensions. In two dimensions, things tend to be better. We know of examples where adding conformal matter and gravity leads to a topological field theory, i.e. a much simpler system. In two dimensions, there are also many exactly integrable models. What happens to them when they are coupled to two-dimensional gravity? To try to get an idea, Ian Kogan and I have investigated the case of the integrable chiral Gross-Neveu model. We did neither prove nor disprove integrability of this model when coupled to gravity, but we obtained some encouraging indications that integrability might indeed be preserved.

In these notes, after introducing the model, I will describe these indications: we checked, up to the second non-trivial order whether the two-particle S-matrix remains elastic in the presence of gravity. It turned out that this is the case.

2. The chiral Gross-Neveu model and the Bethe ansatz

The action of the chiral Gross-Neveu model is given by

$$S \sim \frac{1}{\sqrt{2}} \int \mathrm{d}t\mathrm{d}x \left[-i\bar{\psi}^j \gamma^a \partial_a \psi^j + \frac{1}{2}g^2(\bar{\psi}^j\psi^j)^2 - \frac{1}{2}g^2(\bar{\psi}^j\gamma_5\psi^j)^2 \right]$$
$$= \int \mathrm{d}t\mathrm{d}x \left[i(\psi_+^j)^*\partial_-\psi_+^j + i(\psi_-^j)^*\partial_+\psi_-^j + \sqrt{2}g^2(\psi_+^i)^*(\psi_-^j)^*\psi_-^i\psi_+^j \right] \ . \tag{2.1}$$

(I will detail my conventions in the next section.) One has N colours of Dirac fermions ψ^j ($j = 1, \ldots N$), and ψ_-^j and ψ_+^j refer to right and left-moving components (Weyl fermions).[†] It is often convenient to rewrite the ψ self-couplings using two auxiliary scalar fields σ and π with unit propagator and coupling to the fermions as $\sim g\sigma\bar{\psi}^j\psi^j$ and $\sim ig\pi\bar{\psi}^j\gamma_5\psi^j$. Gross and Neveu [2] computed the effective potential for these scalar fields. To leading order in $\frac{1}{N}$,

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[†] In ref. 1, we mistakenly called ψ_- left-moving and ψ_+ right-moving.

the effective potential for σ (at $\pi = 0$ e.g.) is given by all fermion one-loop diagrams with an arbitrary number of σ's attached. Each individual diagram is IR divergent, but summing them up leads, after renormalisation, to $V_{\text{eff}}(\sigma) = \frac{1}{2}\sigma^2 + \frac{1}{4\pi}g^2N\sigma^2\left[\ln\left(\frac{\sigma}{\sigma^*}\right) - 3\right]$, where σ^* is some fixed subtraction value. This effective potential has a non-trivial minimum at non-zero σ. Hence, σ develops a non-vanishing expectation value, $<\sigma> = \sigma^* e^{1-\pi/Ng^2}$ which in turn, through the $\sigma\bar{\psi}\psi$-interaction, induces a mass $M \sim e^{-\pi/Ng^2}$ for the fermions. Clearly, this dynamical mass generation is non-perturbative.

An alternative way to understand the mass generation is via the Bethe ansatz which, of course, also proves the integrability of the model [3]. Let me very briefly sketch the procedure. One starts with the Hamiltonian that corresponds to the action (2.1). This Hamiltonian is then diagonalized in a n-particle Fock space. At this point there is *no* Dirac sea. The diagonalisation of the Hamiltonian provides the energy levels of the empty Dirac sea (cut off at some large negative energy since n is finite). The eigenstates are pseudoparticles and correspond to the fermions of (2.1) with zero mass. One has a many-body theory of massless pseudoparticles. Then one fills this (cut-off) Dirac sea with the pseudoparticles up to some Fermi level. The physical spectrum is given by particle-hole type excitations. Suppose one takes one of the pseudoparticles out of the filled sea. This creates a hole. But in addition, all other pseudoparticles in the sea are affected by this hole creation. One can show that all $\frac{n}{2}$ right-moving pseudoparticles have their energy and momentum changed by $\Delta E = \Delta k \sim \frac{a}{n}$, while all left-moving ones suffer a change $\Delta E = -\Delta k \sim \frac{b}{n}$ for some constants a, b. Parametrizing the latter as $a = Me^{\theta}$ and $b = Me^{-\theta}$ the total change then is

$$\Delta E_{\text{tot}} = \frac{n}{2}\left(\frac{a}{n} + \frac{b}{n}\right) = M\cosh\theta$$

$$\Delta k_{\text{tot}} = \frac{n}{2}\left(\frac{a}{n} - \frac{b}{n}\right) = M\sinh\theta \ .$$

(2.2)

This is precisely the dispersion relation for a massive excitation.[‡]

Using the Bethe ansatz, it has been shown[3,4] that the S-matrix for pseudoparticle scattering is factorisable and elastic. Factorisability means that any $m \to l$ scattering process can be described by products of $2 \to 2$ S-matrix elements. Elasticity means that in any such

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[‡] Of course, there is also the hole energy and momentum which still obey a massless dispersion relation and which decouple [4].

scattering process only internal quantum numbers (e.g. colour) are exchanged between the particles while they keep their individual momenta and energies. Since physical excitations are made up from pseudoparticle configurations, it could also be shown [3] that the S-matrix for the scattering of the physical excitations, i.e. of the physical (massive) particles, also is elastic and factorisable, as a consequence of these same properties for the pseudoparticle S-matrix.

3. Coupling to gravity

To couple the Gross-Neveu model to two-dimensional gravity, we replace the action (2.1) by its generally covariant form[§]

$$S_{\mathrm{M}} = \frac{1}{\sqrt{2}} \int \mathrm{dt}\mathrm{d}x (\det e) \left[-i\bar{\psi}^j \gamma^a e_a{}^\mu \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{cd}\gamma_{cd} \right) \psi^j + \frac{1}{2}g^2(\bar{\psi}^j\psi^j)^2 - \frac{1}{2}g^2(\bar{\psi}^j\gamma_5\psi^j)^2 \right] \tag{3.1}$$

where $e_a{}^\mu$ is the inverse zweibein, $\omega_\mu{}^{cd}$ the spin-connection and $\gamma_{cd} = \frac{1}{2}[\gamma_c, \gamma_d]$. It is well-known that for two-dimensional Majorana spinors the spin-connection drops out. Here however, the ψ^j are Dirac spinors and we need to introduce the $\omega_\mu{}^{cd}$. We also add the gravity action

$$S_{\mathrm{G}} = -\frac{\gamma}{16\pi} \int \mathrm{dt}\mathrm{d}x \sqrt{-g} R \frac{1}{\nabla^2} R + \mu \int \mathrm{dt}\mathrm{d}x \sqrt{-g} . \tag{3.2}$$

The latter is the well-known induced gravitational action as first derived by Polyakov [5] (see also section 2 of the second ref. 1). It can be interpreted as due to the metric dependence of the ghost and matter measures in the functional integral. The value of the prefactor γ will be given below.

In conformal gauge the gravitational action (3.2) reduces to the Liouville action. Here instead, we will work in chiral (or light-cone) gauge where the metric and zweibein take the

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§ Our conventions are: $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$, $\partial_\pm = \frac{1}{\sqrt{2}}(\partial_0 \pm \partial_1)$, $x_a y^a = x_+ y^+ + x_- y^-$. The Minkowski metric is $\eta_{00} = -1$, $\eta_{11} = 1 \Rightarrow \eta_{+-} = \eta^{+-} = -1$, and $g_{\mu\nu} = e_{a\mu}e^a{}_\nu$, and $e_a{}^\mu e^a{}_\nu = \delta^\mu_\nu$ where, as usual, Lorentz indices $(a, b, \ldots)$ are raised and lowered with η_{ab} while $g_{\mu\nu}$ is used for curved space indices $(\mu, \nu, \ldots)$. One defines $\bar{\chi} = \chi^+ \gamma^0 = \chi \gamma^0$ and $\gamma^0 \gamma^1 = \gamma_5$ so that $\bar{\chi}\gamma^0 = -\chi$ and $\bar{\chi}\gamma^1 = \chi\gamma_5$. Furthermore $(\gamma^0)^+ = -\gamma^0, (\gamma^1)^+ = \gamma^1, \gamma_5^+ = \gamma_5$ and $\chi_\pm = \frac{1}{2}(1 \pm \gamma_5)\chi$, so that $\bar{\chi}\gamma^\pm = \frac{1}{\sqrt{2}}\bar{\chi}(\gamma^0 \pm \gamma^1) = -\sqrt{2}\bar{\chi}_\mp$. Finally note that for a 2×2-matrix M one has $\det M = M_{00}M_{11} - M_{01}M_{10} = M_{++}M_{--} - M_{+-}M_{-+}$.

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form

$$ds^2 = -2dx^+dx^- - 2h_{++}(dx^+)^2$$
$$e_{+-}e_{-+} = 1 \, , \quad e_{--} = 0 \, , \quad \frac{e_{++}}{e_{+-}} = h_{++} \, . \tag{3.3}$$

We are free to make a convenient choice of the Lorentz phase such that $e_{+-} = -1$. Then $e_{-+} = -1$, too, and $e_{++} = -h_{++}$. It is then straightforward to compute the spin-connection from $\omega_\mu{}^{ab} = -\omega_\mu{}^{ba}$ and $\partial_{[\mu}e_{\nu]}{}^a + \omega_{[\mu}{}^{ab}e_{\nu]}{}^c\eta_{bc} = 0$. One readily finds $\omega_+{}^{+-} = \partial_-h_{++}$ and $\omega_-{}^{+-} = 0$, so that $\frac{1}{4}\omega_+{}^{cd}\gamma_{cd} = \frac{1}{2}\partial_-h_{++}\gamma_5$ while $\frac{1}{4}\omega_-{}^{cd}\gamma_{cd} = 0$. Finally the matter action takes the form

$$S_{\mathrm{M}} = \int \mathrm{d}t\mathrm{d}x \Big[i\psi_-^{j}{}^* \Big(\partial_+ - h_{++}\partial_- - \frac{1}{2}\partial_-h_{++} \Big) \psi_-^j + i\psi_+^{j}{}^*\partial_-\psi_+^j$$
$$+ \sqrt{2}g^2(\psi_+^i)^*(\psi_-^j)^*\psi_-^i\psi_+^j \Big] \, . \tag{3.4}$$

The contribution from the spin connection is of course just what one needs to be able to rewrite this action in the more symmetric form

$$S_{\mathrm{M}} = \int \mathrm{d}t\mathrm{d}x \Big[i\psi_-^{j}{}^*\partial_+\psi_-^j + i\psi_+^{j}{}^*\partial_-\psi_+^j + \frac{i}{2}h_{++} \Big(\partial_-\psi_-^{j}{}^*\psi_-^j - \psi_-^{j}{}^*\partial_-\psi_-^j \Big)$$
$$+ \sqrt{2}g^2(\psi_+^i)^*(\psi_-^j)^*\psi_-^i\psi_+^j \Big] \, . \tag{3.5}$$

The gravity action (3.2) in chiral gauge becomes

$$S_{\mathrm{G}} = \frac{\gamma}{8\pi} \int \mathrm{d}t\mathrm{d}x(\partial_-^2 h_{++}) \frac{1}{\partial_-(\partial_+ - h_{++}\partial_-)} \partial_-^2 h_{++} + \mu \int \mathrm{d}t\mathrm{d}x \, . \tag{3.6}$$

From these expressions one sees that the matter action splits into a h_{++}-independent part that is exactly the same as in eq. (2.1) plus a term $\sim h_{++} \Big(\partial_-\psi_-^{j}{}^*\psi_-^j - \psi_-^{j}{}^*\partial_-\psi_-^j \Big)$ which provides an interaction between the right-moving fermions and the "graviton" (h_{++}). The left-moving fermions ψ_+^j do not interact with gravity. In the gravitational action the second term does not depend on h_{++} and hence does not contribute to the dynamics. So we will drop it in the following. The first term, however, does depend on h_{++} in a non-polynomial way. If we use $\frac{1}{\gamma}$ as a formal expansion parameter (i.e. $h_{++} \sim \frac{1}{\sqrt{\gamma}}$) one can expand the $(\partial_+ - h_{++}\partial_-)^{-1}$ in a power series leading to

$$S_{\mathrm{G}} = \frac{\gamma}{8\pi} \int \mathrm{d}t\mathrm{d}x h_{++}\partial_- \frac{1}{\partial_+} \big[1 + \partial_+^{-1}h_{++}\partial_- + (\partial_+^{-1}h_{++}\partial_-)^2 + \ldots \big] \partial_-^2 h_{++} \, . \tag{3.7}$$

from which one can read of the h_{++} ("graviton")-propagator, as well as the various vertices involving an arbitrary number of "gravitons".

The graviton propagator is given by

$$< h_{++}(-k)h_{++}(k) > = -\frac{4\pi i}{\gamma}\frac{k_+}{k_-^3} \qquad (3.8)$$

with an appropriate $i\epsilon$-prescription to be discussed below, while the vertex e.g. between three gravitons of momenta p, q and $k = -p - q$ is

$$i\frac{\gamma}{4\pi}\left[p^2 q^2 + p^2 k^2 + q^2 k^2\right] \ . \qquad (3.9)$$

From the matter action (3.5) one reads the fermion propagators $\frac{i}{k_-}$ for the left-moving fermion (ψ_+) and $\frac{i}{k_+}$ for the right-moving fermion (ψ_-). Most important for us here is the vertex between an (incoming) right-moving fermion of momentum p, an (outgoing) right-moving fermion of momentum p' and an (incoming) graviton of momentum $k = p' - p$ which is

$$-\frac{i}{2}(p_- + p'_-) \ . \qquad (3.10)$$

Of course, there is also the four-fermion vertex that was already there before coupling to gravity. It is $i\sqrt{2}g^2$ between an (incoming) left-moving fermion of colour j, an (incoming) right-moving fermion of colour i and an (outgoing) left-moving fermion of colour i and an (outgoing) right-moving fermion of colour j.

4. The vanishing of the gravitational corrections to the S-matrix

Our goal now is to compute the two pseudoparticle S-matrix in the presence of gravity and show that it is still elastic. As explained above, the pseudoparticles are the massless fermions described by the action (3.5), while the physical particles are the massive fermions. The pseudoparticles correspond to an *empty* Dirac sea. This means that one cannot create a pseudoparticle-antipseudoparticle pair. Accordingly, the pseudoparticle propagators $\frac{i}{k_\pm}$ read from (3.5) do *not* have the standard Feynman $i\epsilon$-prescription which would be $\frac{ik_\mp}{k_+k_-+i\epsilon}$. Instead, we must only use the retarded propagator

$$< \psi_\mp^{l}{}^{*}(-k)\psi_\mp^{m}(k) > = \ \delta_{lm}\frac{i}{k_\pm + i\epsilon} \ = \ \delta_{lm}\frac{ik_\mp}{k_+k_- + i\epsilon\,\mathrm{sgn}k_\mp} \ . \qquad (4.1)$$

An important consequence of the appearence of the retarded fermion propagators only, and of the structure of the four-fermion vertex, is that the diagrams of Fig. 1 do contribute to
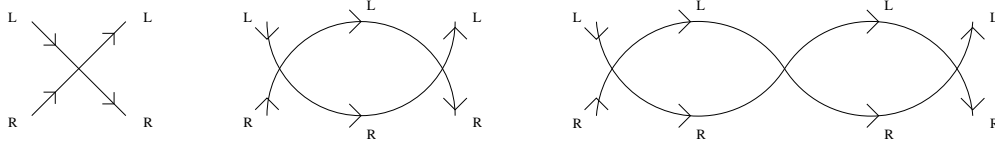
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Fig. 1: Diagrams contributing to the left-right scattering

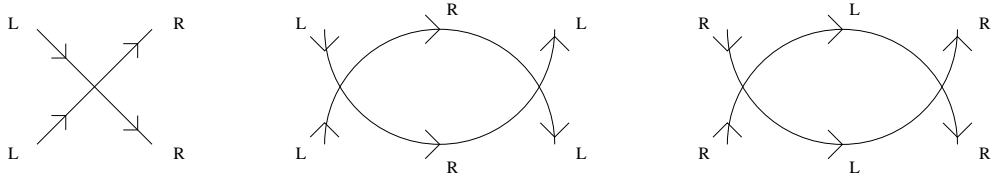pseudoparticle scattering amplitudes, while those of Figs. 2 and 3 do not exist or give vanishing contribution.

Fig. 2: Left-left and right-right scattering diagrams giving vanishing contributions
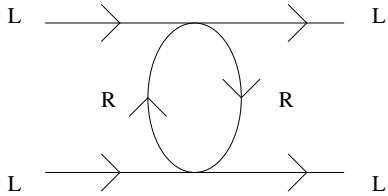

Fig. 3: Another left-left scattering diagram giving a vanishing contribution

Thus, without gravity, there is no left-left and no right-right scattering. The only non-trivial scattering is left-right $\rightarrow$ left-right, corresponding to diagrams as in Fig. 1. How does the latter manage to be elastic, i.e. preserve the individual momenta? This is a simple consequence of energy-momentum conservation and the mass-shell condition. Indeed, let the initial momenta be p and q and the final momenta p' and q' with p, p' for the left-moving and q, q' for the right-moving fermions. The conservation condition then is

$$p_+ + q_+ = p'_+ + q'_+ \ , \quad p_- + q_- = p'_- + q'_- \ . \tag{4.2}$$

The mass-shell conditions are

$$p_- = p'_- = 0 \ , \quad q_+ = q'_+ = 0 \ . \tag{4.3}$$

Combining both equations gives

$$p_+ = p'_+ , \quad q_- = q'_- \tag{4.4}$$

which together with (4.3) shows that the individual momenta are unchanged and the two pseudoparticle S-matrix is elastic in the left-right $\rightarrow$ left-right chanel. Of course, this applies only to the S-matrix; general off-shell amplitudes do not preserve the individual momenta.
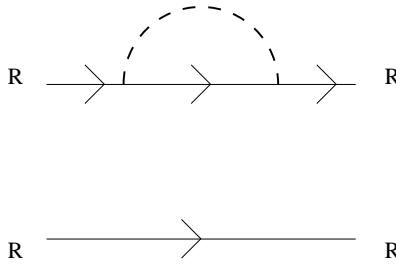

Fig. 4: Disconnected diagram contributing only to the gravitational self-energy of the right fermions
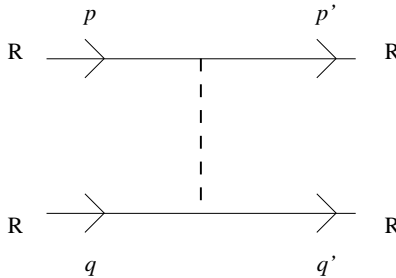

Fig. 5: Gravitational tree-level scattering of two right fermions

What changes when we couple this system to gravity? Now the right-moving fermions couple to the graviton with the vertex (3.10). This will affect the left-right $\rightarrow$ left-right correlation functions, but by the above kinematic argument the corresponding S-matrix element will remain elastic. Also, the left-left $\rightarrow$ left-left scattering is insensitive to gravity and thus remains elastic, too. The only non-trivial case is the right-right $\rightarrow$ right-right scattering. There is no kinematic reason for this to remain elastic, and the fermions do couple to gravity via diagrams as shown in Figs. 4 - 7. Of course, the diagram of Fig. 4 cannot lead to any non-trivial

scattering. The first diagram that could do so is the tree-diagram of Fig. 5 involving a one-graviton exchange between two right-moving fermions. Recall from eq. (3.8) that the graviton propagator is $-\frac{4\pi i}{\gamma}\frac{k_+}{k_-^3}$. Contrary to the fermions, it should be the Feynman propagator and have the standard $i\epsilon$-prescription, i.e.

$$-\frac{4\pi i}{\gamma}\frac{k_+}{k_-^3} \;\rightarrow\; -\frac{4\pi i}{\gamma}\frac{k_+^4}{(k_+k_- + i\epsilon)^3} = -\frac{4\pi i}{\gamma}\frac{k_+}{(k_- + i\epsilon\,\mathrm{sgn}k_+)^3} \;. \tag{4.5}$$

Let's first compute the diagram of Fig. 5. Actually, we will compute the amputated diagram on shell as relevant for the S-matrix. It is given by

$$\frac{i\pi}{\gamma}(p_- + p_-')(q_- + q_-')\frac{(p_+ - p_+')}{(p_- - p_-')^3} \;. \tag{4.6}$$

Putting the external momenta on shell, $p_+ = p_+' = q_+ = q_+' = 0$ for right-moving fermions, the amplitude (4.6) vanishes. It is clear from the structure of the graviton propagator that one always gets zero as long as it is coupled to two on-shell right-moving fermions. It then immediately follows that both diagrams of Fig. 6 also vanish on shell.
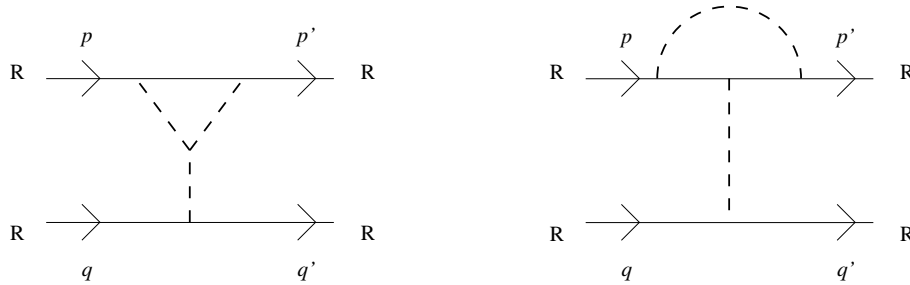

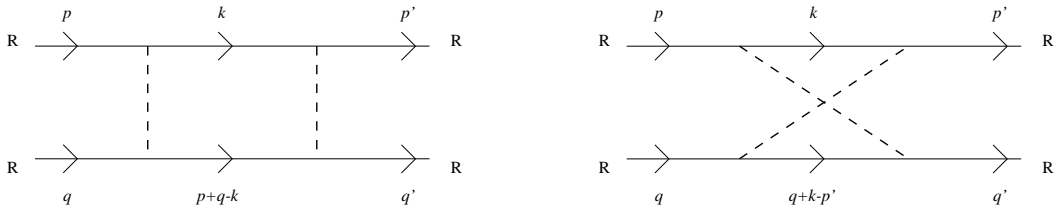**Fig. 6:** Vertex corrections to the diagram of Fig. 5

Fig. 7: The box and crossed box diagrams contributing to the one-loop gravitational scattering of two right fermions

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So, up to order $\frac{1}{\gamma^2}$, we are left with the two diagrams of Fig. 7 only. They yield

$$
\frac{1}{4\gamma^2} \int \mathrm{d}^2k \; \frac{(p_- + k_-)(p'_- + k_-)(p_+ - k_+)(k_+ - p'_+)}{k_+(p_- - k_-)^3(k_- - p'_-)^3} \times
$$
$$
\times \left[\frac{(2q_- + p_- - k_-)(2q'_- + p'_- - k_-)}{q_+ + p_+ - k_+} + \frac{(2q_- - p'_- + k_-)(2q'_- - p_- + k_-)}{q_+ - p'_+ + k_+} \right] . \tag{4.7}
$$

Again, we only want to evaluate it on-shell where it reduces to

$$
-\frac{(q_- + q'_-)}{2\gamma^2} \int \mathrm{d}^2k \; \frac{(k_- + p_-)(k_- + p'_-)(2k_- - p_- - p'_-)}{(k_- - p_-)^3(k_- - p'_-)^3} \; . \tag{4.8}
$$

Although it looks as if the integrand does not depend on k_+, this is not true. The k_+-dependence comes in through the $i\epsilon$-term in the graviton propagators. Let's integrate over k_- first. The integrand falls off fast enough at infinity so that one can close the contour by a semicircle in the upper or lower half-plane. We have two third-order poles (assuming $p_- \neq p'_-$ i.e. $p \neq p'$; if $p = p'$ we have a pole of order six and the integral obviously vanishes). According to the $i\epsilon$-prescription (4.5) they are at $k_- = p_- - i\epsilon \, \mathrm{sgn}(k_+ - p_+)$ and $k_- = p'_- - i\epsilon \, \mathrm{sgn}(k_+ - p'_+)$. If we choose the semicircle in the lower half-plane, we will pick up residues from the first pole only if $k_+ > p_+$ and from the second pole only if $k_+ > p'_+$. One gets

$$
-\frac{2\pi i}{\gamma^2} \frac{(q_- + q'_-)(p_- + p'_-)}{(p_- - p'_-)^3} \int \mathrm{d}^2k \left[\theta(k_+ - p_+) - \theta(k_+ - p'_+) \right]
$$
$$
= \frac{2\pi i}{\gamma^2} \frac{(q_- + q'_-)(p_- + p'_-)(p_+ - p'_+)}{(p_- - p'_-)^3} \; . \tag{4.9}
$$

Had we chosen the semi-circle in the upper half-plane, the result would have been the same, of course. Thus the one-loop result (on shell) is, up to the factor $\frac{2}{\gamma}$, identical to the tree-level result, and hence vanishes for the same reason ($p_+ = p'_+ = 0$ on shell).

Let me stress that the vanishing of the scattering amplitude at one loop was in no way obvious a priori. One has to add both diagrams of Fig. 7 and use the on shell condition. Then, it is only through the subtlety of the $i\epsilon$-prescription that one gets the factor $(p_+ - p'_+)$ which makes the amplitude vanish on shell. It is very tempting to speculate that this sort of mechanism will persist at all orders in $\frac{1}{\gamma}$.

5. Conclusions

We have seen that, in general, the coupling to gravity does modify the right-right → right-right correlation functions. However, the gravitational corrections to the *on shell* right-right → right-right scattering amplitude vanish at tree and one-loop level, i.e. up to and including order $\frac{1}{\gamma^2}$. We thus conjectured [1] that this might be true to all orders in $\frac{1}{\gamma}$. Let's assume here that this is indeed the case, and see what it implies.

First of all, one might object the use of perturbation theory in $\frac{1}{\gamma}$. The constant γ is given by [5]

$$\gamma = \frac{1}{12}\left(c - 13 - \sqrt{(c-1)(c-25)}\right)$$

where c is the total matter central charge. Of course, gravity is well understood for $c \leq 1$ where γ is real. This is the weak coupling regime. Indeed, as $c \to -\infty$ one has $\gamma \sim \frac{c}{6}$. Since $\frac{1}{\gamma}$ is the gravitational coupling constant, $c \to -\infty$ is the gravitational weak-coupling limit. A perturbation expansion in $\frac{1}{\gamma}$ could be expected to be reasonable as long as c is large and negative. In the Gross-Neveu model this is certainly not the case. Moreover, in ref. 1 we have shown that, in the presence of gravity, certain fermion correlation functions have a diverging expansion in $\frac{1}{\gamma}$ for all γ, and that the Borel-resummed perturbation series exhibits a typically non-perturbative behaviour. Here, however, the situation is different: the perturbative expansion of the right-right → right-right on shell scattering amplitude, having only zero coefficients, converges everywhere in the complex $\frac{1}{\gamma}$-plane, yielding zero for all γ. Although one cannot exclude a non-perturbative contribution $\sim e^{-a\gamma}$ to the S-matrix, this does not seem to be very likely.

So if the gravitational corrections to the right-right → right-right (pseudoparticle) S-matrix elements do indeed vanish, as they do for the left-left → left-left elements, the only corrections are to the left-right → left-right (pseudoparticle) S-matrix elements. But as already noted earlier, the latter nevertheless remain elastic, and one would conclude that the two pseudoparticle scattering S-matrix remains elastic in all chanels when the coupling to gravity is included. One could then go on and speculate that all S-matrix elements for multi-pseudoparticle scattering factorize, and hence reduce to products of two-pseudoparticle scattering S-matrices, which are all elastic. Thus the complete S-matrix for pseudoparticle scattering would be factorisable and elastic. Since the physical S-matrix for the scattering of the physical (massive) fermions

is obtained from the pseudoparticle S-matrix, it probably would turn out to be elastic and factorisable, too. If this chain of hypothesis goes through, the chiral Gross-Neveu model would indeed remain integrable when coupled to gravity. However, there is still a long way to go.

Note Added : Let me note that after our original papers [1] and at about the same time as my lecture at the Cargese summer school, there appeared a paper by Abdalla and Abdalla [6] where the gravitational dressing of integrable models was studied in conformal gauge. In particular, for the Gross-Neveu model they claimed that coupling to gravity does not invalidate the existence of higher conservation laws, and they concluded that the model remained integrable, thus confirming our conjecture.

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